AP Statistics | Activity 0905 | Name:
Dr. Paul L. Bailey | Tuesday, September 5, 2023

Problem 1. A bin contains three red balls, five green balls, and k blue balls. Three balls are drawn at random from the bin. Find the smallest integer k so that the probability of picking three blue balls equals or exceeds the probability of picking one ball of each color.

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Solution. The sample space has size

$$|S| = \binom{8+k}{3} = \frac{(k+6)(k+7)(k+8)}{6}.$$

But we don't actually need this to solve this problem.

Let E_1 be the event of three blue balls, and E_2 be the event of one ball of each color. Then

$$|E_1| = \binom{k}{3} = \frac{(k-2)(k-1)k}{6} \quad \text{and} \quad |E_2| = 15k.$$

We wish to solve $|E_1| \geq |E_2|$, that is,

$$\frac{(k-2)(k-1)k}{6} \geq 15k.$$

This reduces to $k^2 - 3k + 2 \geq 90$, so $k^2 - 3k - 88 \geq 0$, that is, $(k+4)(k-11) \geq 0$. Thus, $k = 11$. □